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Radiation and Soret Effect on Visco-Elastic MHD Oscillatory Horizontal Channel Flow With Heat and Mass Transfer in Presence of Chemical Reaction and Heat Source

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ABSTRACT

An analysis of radiation and soret effect on visco-elastic mhd oscillatory flow with heat and mass transfer through a porous medium bounded by two infinite horizontal parallel porous plates in presence of chemical reaction and a heat source has been presented when one plate is kept at rest while the other is oscillating in its own plane. The fluid is considered to be non-Newtonian characterized by Walters liquid (Model B'). The temperature of the stationary plate is assumed to be constant whereas the temperature of the other plate varies periodically with time about a steady mean. The equations governing the fluid flow, heat and mass transfer have been solved analytically. The expressions for velocity, temperature, species concentration, non-dimensional skin-friction at the plates, the coefficient of rate of heat transfer from the plates to the fluid in terms of Nusselt number in non-dimensional form, the coefficient of rate of mass transfer from the plates to the fluid in terms of Sharewood number in non-dimensional form are obtained and illustrated graphically to observe the visco-elastic effects in combination of other flow parameters involved in the solution. It is noticed that the momentum, thermal and concentration fields are significantly affected by the visco-elastic parameter.

Keywords and phrases: MHD, visco-elastic, oscillatory flow, skin-friction, Nusselt number, Share wood number, Radiation, Soret number.

I. INTRODUCTION

The free convective Visco-elastic mhd flow with heat and mass transfer has been studied in a large scale due to its application in various fields like, cosmic fluid dynamics, meteorology, solar physics, in the motion of the core of the earth plasma studies, nuclear reactors, geothermal energy extractions and the boundary layer control in the field of aerodynamics. The attention of a large number of scholars has been attracted by the heat and mass transfer problems in combination with chemical reaction because of their great importance in many processes. An important role is played by MHD in the field of Astrophysics, Geophysics and many engineering problems. Nowadays, the studies of fluid flows through porous medium become interesting and inevitable in case of extraction of crude oil from the pores of rocks. A reaction is said to be of the order n, if the reaction rate is proportional to the n-power of concentration. In particular, a reaction is said to be first-order, if the rate of reaction is directly proportional to concentration itself. In well mixed system, the reaction is heterogeneous if it takes place at an interface and homogeneous, if it takes place in The remarkable contributions of the solution. authors like, Alfven [4], Cowling [26], Cramer and Pai [27], Frerraro and Plumton[31] and Shercliff[43] have brought MHD to its present

position. Recently, the study of MHD flow with heat and mass transfer becomes more interesting because of the effect of magnetic field on the performance of various systems. Nigam and Singh[37]. Soundalgekar and Bhat[48]. Vajravelu[51]and Attia and Kotb[8] have made a series of investigations for such applications . Bodosa and Barthakur [9] had studied MHD flow and heat transfer of a Visco-elastic fluid past between two horizontal plates with heat sources or sinks. Raptis et al. [39] has analysed hydro magnetic free convective flow through porous medium between parallel plates. Makinde and Mhone[34]had investigated MHD flow in a channel fluid with porous medium. The combined effects of transverse magnetic field and ohmic dissipation on unsteady flow of a conducting flow through a channel filled with saturated porous medium and non-uniform wall temperature was investigated by Choudhury et al. [14]. Ahmed and Barman [2] have analysed the combined effect of a transverse magnetic field of uniform strength and the velocity of the moving plate in the heat transfer and flow of an electrically conducting fluid bounded by two horizontal plates of which one plate is at rest and the other is oscillating in its own plane.

Singh *et al.* [44] have analysed heat transfer over stretching surface in porous media in

presence of magnetic field. Singh et al. [45] have investigated MHD oblique stagnation point flow towards a stretching sheet with heat transfer. The effect of thermal radiation and magnetic field on unsteady stretching permeable sheet in presence of free stream has been studied by Singh et al. [46]. Elbashbeshv et al. [30] have investigated heat transfer over an unsteady porous stretching surface embedded in a porous medium with variable heat flux with heat source or sink. The problem of heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous mediumheve been studied by Angirasa et al. [5]. Devi and Ganga [28] have investigated the dissipation effects on MHD non linear flow and heat transfer past a porous surface with prescribed heat flux. The problem of mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium have been analysed by Reddy and Reddy [40].

The problems of heat and mass transfer in combination with chemical reaction have played an important role in many processes and have attracted the researchers in a large scale. A reaction is said to be of the order n, if the reaction rate is proportional to the n-power of concentration. In particular, a reaction is said to be first-order, if the rate of reaction is directly proportional to concentration itself. In well mixed system, the reaction is heterogeneous if it takes place at an interface and homogeneous, if it takes place in Muthucumaraswamy solution. and Meenakshisundaram [36] have investigated the effect of chemical reaction on free convection and heat transfer past an oscillating infinite vertical plate . Anjalidevi and Kandasamy [6] have investigated the effect of chemical reaction on MHD flow with heat and mass transfer past a semiinfinite plate. Choudhury and Jha [15] have studied the same on MHD micropolar fluid flow in slip flow regime. Al-Odat and Al-Azab [7] have analysed the influence of chemical reaction on transient MHD free convection over a moving vertical plate. The influence of chemical reaction on MHD flow with heat and mass transfer over a vertical stretching sheet in presence of heat source and thermal stratification effect has been studied by Kandasamy et al. [32]. The effect of chemical reaction on transient MHD free convective flow over a vertical plate in slip flow regime has been studied by Ahmed [3]. The radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source has been analysed by Bala et al. [10]. Baoku et al. [11] have investigated the influence of thermal radiation on a transient MHD Couette flow through a porous

medium. Basu et al. [12] have analysed the radiation and mass transfer effects on transient free convection flow of dissipative fluid past semiinfinite vertical plate with uniform heat and mass flux. The radiation, heat and mass transfer effects on moving isothermal vertical plate in presence of chemical reaction has been studied hv Muthucumaraswamy and Chandrakala [35]. Rao et al. [38] have studied the chemical effects on an unsteady MHD free convection fluid past a semiinfinite vertical plate embedded in a porous medium with heat absorption. The radiation effect on the flow and heat transfer over an unsteady stretching sheet has been investigated by El-Aziz [29]. Sandeep et al. [41] have found out the effect of radiation chemical reaction on transient MHD free convective flow over a vertical plate through porous medium. Suneetha et al. [50] have studied the radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink.

The applications of the mechanisms of non-Newtonian fluid flows in modern technology and industries have attracted the attention of a large number of researchers. Keeping in view of the applications and the important roles played by the non-Newtonian fluid flow mechanisms in various manufacturing processes, authors like Kelly *et al.* [33], Subhash *et al.* [49], Sonth *et al* [47], Abel *et al.* [1], Choudhury and Mahanta [17], Choudhury and Dey [16], Choudhury and Das [25], Choudhury and Das[18], [19], [20], [21], [22], [23], [24], [25] etc. have studied some problems of physical interest in this field.

The object of the present author is to study the visco-elastic effect on the MHD flow of Walters liquid (Model B') with heat and mass transfer in presence of radiation, chemical reaction and Soret effect bounded by two horizontal plates of which one plate is kept at rest while the other is oscillating in its own plane in presence of heat source.

The constitutive equation for Walters liquid (Model B') is

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e^{ik} \tag{1}$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed coordinate system x^i , v^i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^i_{,m} e^{im} - v^i_{,m} e^{mk} \qquad (2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^{i}_{,k} + v^{k}_{,i} \tag{3}$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau) d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau) d\tau \qquad (4)$$

(5)

 $N(\tau)$ being the relaxation spectrum as introduced by Walter[52, 53]. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

 $\int_{0}^{\infty} \tau^{n} N(\tau) d\tau, \quad n \ge 2$

have been neglected.

II. MATHEMATICAL FORMULATION

The oscillatory flow of a visco-elastic fluid characterized by Walters liquid (Model B') through a porous medium Brinkman Model [15] bounded by two infinite horizontal porous plates separated by a distance h of which one plate is kept at rest while the other is oscillating in its own plane with heat and mass transfer in presence of a heat source, radiation and Soret effect has been considered. A homogeneous chemical reaction of first order of constant rate \overline{K}_2 is assumed to exist between the diffusing species and the fluid. A transverse uniform magnetic field is applied normal to the plate. The temperature of the plate at rest is taken to be constant whereas that of the oscillating one varies periodically with time about a steady mean. We restrict our investigation to the following conditions:

- i. All the fluid properties other than the density in the buoyancy force term are constant.
- ii. The Eckert number is small.
- iii. The induced magnetic field can be neglected as the magnetic Reynolds number is very small.
- iv. The plates are electrically non-conducting.
- v. The frequency of oscillation of the temperature of the upper plate is twice the frequency of oscillation of the plate velocity of the upper plate.
- vi. The frequency of oscillation of the concentration on the upper plate is twice the frequency of oscillation of the plate velocity of the upper plate.



The x- and y-axes are taken along and perpendicular to the stationary plate which is horizontal and is directed into the fluid and the zaxis is taken along the width of the stationary plate

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in a direction perpendicular to the y-axis. Let $\vec{q} = \mathbf{i}\vec{u} + \mathbf{j}\vec{v}$ be the fluid velocity at the point $(\mathbf{\bar{x}}, \mathbf{\bar{y}}, \mathbf{\bar{z}})$ and $\mathbf{\bar{B}} = \mathbf{\bar{B}}_{0}\mathbf{j}$ be the applied magnetic field, \mathbf{i} and \mathbf{j} being the unit vectors along x- and y-axes respectively. As the lengths of the plates are infinite, all the physical quantities except possibly the pressure are assumed to be independent of $\mathbf{\bar{x}}$ and the flow is parallel to x-axis. Under the above assumptions, the governing equations of the fluid flow, heat and mass transfer are as follows: The equation of continuity:

$$\frac{\partial \mathbf{u}}{\partial \mathbf{r}} = 0 \tag{6}$$

The momentum equation:

$$\frac{\partial \alpha}{\partial t} = -\frac{1}{\rho} \frac{\partial F}{\partial x} + \nu \left(\frac{\partial^2 \alpha}{\partial x^2}\right) - \frac{\kappa_0}{\rho} \frac{\partial^2 g}{\partial t \partial g^2} - \frac{\nu \alpha}{\kappa} - \frac{\sigma B_0^* \alpha}{\rho} \quad (7)$$
where $\nu = \frac{\eta_0}{\rho}$.

The energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} = -\frac{\kappa}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial g^2} + \frac{\nu}{c_p} \left(\frac{\partial \alpha}{\partial g}\right)^2 - \frac{\kappa_0}{\rho c_p} \frac{\partial \alpha}{\partial g} \frac{\partial^2 \alpha}{\partial \bar{t} \partial g} + \frac{\sigma \bar{B}_0^2 \alpha^2}{\rho c_p} + Q(\bar{T} - \bar{T}_0) - \frac{\partial q_r}{\partial g}$$

Concentration equation: $\frac{\partial \mathcal{C}}{\partial f} = D_M \frac{\partial^2 \mathcal{C}}{\partial g^2} + \frac{D_M \mathcal{R}^*}{T_M} \frac{\partial^2 T}{\partial g^2} - \overline{K}_2 (\overline{\mathcal{C}} - \overline{\mathcal{C}}_0)$ (9)

The boundary conditions are

$$\overline{y} = 0: \overline{u} = 0, \overline{T} = \overline{T}_0, \overline{C} = \overline{C}_0$$

$$\overline{y} = h: u = \overline{U} e^{i \varpi f}, \overline{T} = \overline{T}_0 + \overline{T}_w e^{2i \varpi f},$$

$$\overline{C} = \overline{C}_0 + \overline{C}_w e^{2i \varpi f}$$

$$(10)$$

where \overline{u} is the axial velocity, \overline{B}_0 is the strength of the applied magnetic field, \overline{T}_0 is the temperature of the plate at rest, \overline{C}_0 is the concentration on the plate at rest, \overline{T}_w the amplitude of the fluctuating part of the temperature of the moving plate, \overline{C}_w is the amplitude of the fluctuating part of the concentration at the moving plate and \overline{U} is the amplitude of the velocity of the moving plate, κ is the thermal conductivity, \overline{K}^* is the thermal diffusion ratio, T_M is the mean temperature of the fluid, D_M coefficient of mass diffusivity, σ is the electrical conductivity and K is the permeability parameter.

We introduce the following nondimensional quantities:

 $x = \frac{x}{h}, \quad y = \frac{y}{h}, \quad \theta = \frac{\tau - \tau_0}{\tau_w}, \quad \phi = \frac{c - c_0}{c_w},$ $M = \frac{\sigma h^2 E_0^2}{\rho v}, \quad t = \frac{f v}{h^2}, \quad w = \frac{w h^2}{y}, \quad u = \frac{\alpha h}{v}, \quad U = \frac{\sigma h}{v},$ $P = \frac{p h^2}{\rho v^2}, \quad S = \frac{h^2}{\kappa}, \quad K_r = \frac{R_2 h^2}{v}, \quad P_r = \frac{\mu c_p}{\kappa} \text{ (Prandtl Number)}, \quad E = \frac{v^2}{c_p h^2 \tau_w} \text{ (Eckert Number)}, \quad \alpha = \frac{q h^2}{v}$

(Heat source parameter). $S_{\tau} = \frac{D_M T_W R^*}{v T C_W}$ (Soret Number).

The Roseland approximation, quantified the radiative heat flux for an optically thick boundary layer flow in a simplified differential form is considered as

$$\bar{q}_r = -\frac{4\sigma^*}{3k^*}\frac{\partial\bar{T}^*}{\partial y}$$

where σ^* is the Stefan Boltzmann constant and k^* is the mean absorption coefficient. Expanding \overline{T}^4 in Taylor's series about \overline{T}_0 , which after neglecting higher order terms, takes the form $\overline{T}^4 = 4\overline{T}_0^{\ 3}\overline{T} - 3\overline{T}_0^{\ 4}$.

Equation (7) becomes,

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 y}{\partial y^2} - K_1 \frac{\partial^3 y}{\partial t \partial y^2} - (S + M)u \qquad (11)$$
Where $K_1 = \frac{K_0}{\rho h^2}$.

Equation (8) becomes,

$$\frac{\partial \theta}{\partial t} = -\frac{1}{p_r} \left(1 + \frac{1}{R} \right) \frac{\partial^2 \theta}{\partial y^2} + E \left(\frac{\partial u}{\partial y} \right)^2 - K_1 E \frac{\partial u}{\partial y} \frac{\partial^2 y}{\partial t \partial y} + ME u^2 + \alpha \theta$$

(12) Where $R = \frac{3\kappa K^*}{16\sigma^* T_0^3}$ = Radiation parameter. Equation (9) becomes,

$$\frac{\partial \phi}{\partial t} = \frac{1}{s_c} \frac{\partial^2 \phi}{\partial y^2} + S_r \frac{\partial^2 \phi}{\partial y^2} - K_r \phi \tag{13}$$

The boundary conditions (Non-dimensional) are: $y = 0: u = 0, \theta = 0, \phi = 0$ $y = 1: u = Ue^{i\omega t}, \theta = e^{2i\omega t}, \phi = e^{2iwt}$ (14)

III. METHOD OF SOLUTION

We assume that

$$\frac{\partial P}{\partial x} = Ae^{i\omega t}$$
.

Then equation (10) becomes

$$\frac{\partial u}{\partial t} = -Ae^{i\omega t} + \frac{\partial^2 u}{\partial y^2} - K_1 \frac{\partial^3 y}{\partial t \partial y^2} - (S + M)u \quad (15)$$

In view of the boundary conditions (12), we assume the velocity, temperature and concentration fields as

$$u = u_0(y)e^{i\omega t}, \ \theta = \theta_0(y)e^{2i\omega t}, \ \phi = \phi_0(y)e^{2i\omega t}$$
(16)

Using (16), equations (11), (12) and (13) reduce to

$$u_{0}^{"} - \frac{P_{r}}{1 - K_{1}i\omega} u_{0} = \frac{P_{r}}{1 - K_{1}i\omega}$$
(17)
$$\theta_{0}^{"} - \frac{P_{r}}{1 + \frac{1}{R}} (2i\omega - \alpha)\theta_{0} = \frac{EP_{r}}{1 + \frac{1}{R}} (1 - K_{1}i\omega) u_{0}^{'2} - \frac{P_{r}ME}{1 + \frac{1}{R}} u_{0}^{2}$$
(18)

And

$$\phi_0'' - S_c (K_r + 2iw) \phi_0 = -S_c S_r \phi_0'$$
(19)

The boundary conditions (14) are now,

y=0: $u_0=0$, $\theta_0=0$, $\phi_0 = 0$ y=1: $u_0=U$, $\theta_0=1$, $\phi_0 = 1$ (20) Solving equations (17), (18) and (19) subject to the boundary conditions (20) we get the solutions as: $u_0 = C_1 e^{a_1 y} + C_2 e^{-a_1 y} + a_2$ (21) $\theta_0 = C_2 e^{a_3 y} + C_4 e^{-a_3 y} + a_9 e^{2a_1 y} + a_{10} e^{-2a_1 y} +$

$$a_{11}e^{-1y} + a_{12}e^{-1y} + a_{13}$$
(22)

$$\phi_0 = C_5e^{a_{16}y} + C_6e^{-a_{16}y} + a_{17}e^{a_3y} + a_{19}e^{-a_3y} + a_{19}e^{2a_1y} + a_{20}e^{-2a_1y} + a_{21}e^{a_1y} + a_{22}e^{-a_1y}$$
(23)

We write

$$\begin{aligned} u_0(y) &= u_{0r} + iu_{oi}, \theta_0(y) = \theta_{0r} + i\theta_{oi}, \\ \phi_0(y) &= \phi_{0r} + i\phi_{oi} \\ \text{So that} \\ u &= u_0(y)e^{i\omega t}, \ \theta = \theta_0(y)e^{2i\omega t}, \ \phi = \phi_0(y)e^{2i\omega t} \\ \text{become} \\ u &= (u_{0r} + iu_{0i})(\cos\omega t + i\sin\omega t) \\ \theta &= (\theta_{0r} + i\theta_{0i})(\cos2\omega t + i\sin2\omega t), \\ \phi &= (\phi_{0r} + i\phi_{0i})(\cos2\omega t + i\sin2\omega t). \end{aligned}$$

Considering the real part only, we get the nondimensional velocity, temperature and concentration field u as

$$u = u_{0r} cos\omega t - u_{oi} sin\omega t \tag{24}$$

$$\theta = \theta_{0r} \cos i\omega t - \theta_{0i} \sin 2\omega t \tag{25}$$

$$\phi = \phi_{0r} \cos 2\omega t - \phi_{0i} \sin 2\omega t \tag{26}$$

The non-dimensional skin friction is given by $\partial u = \frac{\partial u}{\partial u} \frac{\partial^2 u}{\partial u}$

$$\sigma_{xy} = \frac{1}{\partial y} - K_1 \frac{1}{\partial t \partial y}$$

The non-dimensional skin friction at the plates y=0 and y=1 in the direction of the flow are given by $\sigma_0 = Re(\sigma_{xy})_{y=0}$,

$$\sigma_{1} = Re(\sigma_{xy})_{y=1}^{y=0}$$

i.e.

$$\sigma_{0} = \{u_{0r}'(0) - K_{1}\omega u_{0i}'(0)\}cos\omega t + \{K_{1}\omega u_{0r}'(0) - u_{oi}'(0)\}sin\omega t$$

$$\sigma_{1} = \{u_{0r}'(1) - K_{1}\omega u_{0i}'(1)\}cos\omega t + \{K_{1}\omega u_{0r}'(1) - u_{oi}'(1)\}sin\omega t$$

$$(28)$$

The non-dimensional form of the rate of heat transfer at the plates y=0 and y=1 in the form of Nusselt number N_u are given by,

$$N_{u_0} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0}, \quad N_{u_1} = \left(\frac{\partial \theta}{\partial y}\right)_{y=1}.$$

So that

$$N_{u_0} = \theta_{0r}'(0)\cos 2\omega t - \theta_{0i}'(0)\sin 2\omega t$$
(29)

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$$N_{u_1} = \theta_{0r}'(1)\cos 2\omega t - \theta_{0i}'(1)\sin 2\omega t$$
(30)
respectively.

The non-dimensional form of the rate of mass transfer at the plates y=0 and y=1 in terms of Sherwood number S_h are given by

$$S_{h_0} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0}, \quad S_{h_1} = \left(\frac{\partial \phi}{\partial y}\right)_{y=1}.$$

 $S_{h_0} = \phi_{0r}'(0) \cos 2\omega t - \phi_{0i}'(0) \sin 2\omega t$ (31)

$$S_{h_1} = \phi_{0r}'(1)\cos 2\omega t - \phi_{0i}'(1)\sin 2\omega t$$
(32)

respectively.

The constants are obtained but not given here due to brevity.

IV. DISCUSSION

The purpose of the present study is to find out the visco-elastic effects on MHD flow with heat and mass transfer of Walters liquid (Model B') through a porous medium bounded by two infinite horizontal parallel porous plates in presence of radiation, chemical reaction, Soret effect and heat source when one plate is kept at rest while the other is oscillating in its own plane. The visco-elastic effect is characterized by the non zero values of the non dimensional parameter K_1 whereas K_1 =0 represents the Newtonian fluid flow phenomenon.

To understand the physics of the problem, the physical quantities such as, the velocity profile, the skin-friction coefficient, the temperature profile, the Nusselt number, the Sharewood number and the concentration profile are depicted for the cases when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1). All the numerical calculations are to be carried out for α =.05, E=.01, ω =3, A=1, S=.1, t=.1, S_c=.1, P_r=3 throughout the problem.



Fig-2: α =.05; E=.01; M=1;w=3;U=0; A=1;S=.1;



$$t=.1;R=.1;K_r=.1;S_r=.1;S_r=.1;P_r=3.$$



 $\label{eq:Fig-3: a=.05; E=.01; M=1; w=3; U=1; A=1; S=.1; \\ t=.1; R=.1; K_r=.1; S_r=.1; S_c=.1; P_r=3. \\$

Figure 2 and 3 illustrate that the velocity field of the fluid flow when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively.Fig-2 reveals that the velocity profile is parabolic in nature and maximum value occurs along the centre of the channel whereas minimum values occur at the walls in case of both Newtonian and non-Newtonian fluid cases. Fig-3 depicts an increasing trend of the fluid velocity with the maximum at the plate y=1 and minimum value at the plate y=0. In both the figures, the speed of the fluid velocity enhances with the growth of the visco-elastic parameter K_1 in comparison to that of Newtonian fluid flow.



Fig-4: α =.05; E=.01;w=3;U=0;A=1;S=.1; t=.1;R=.1;K_r=.1;S_r=.1;S_r=.1;y=.5;P_r=3;



Figures 4 and 5 explain the nature of the fluid velocity against the magnetic field. In both the figures, a decreasing trend of the velocity profile is observed against the magnetic parameter M. It is also observed from fig-4 and fig-5 that the velocity profile increases when the amplitude of the velocity of the moving plate is non-zero (U=1) in comparison with the case when the amplitude of the velocity factor enhances the fluid velocity in comparison with that of Newtonian fluid flow.



 $t=.1;R=.1;K_r=.1;S_r=.1;S_c=.1;P_r=3;$



Fig-7: a=.05; E=.01; w=3; U=1;A=1;S=.1 t=.1;R=.1; K_r=.1;S_r=.1;S_c=.1;P_r=3;

Figure 6 illustrates that the nondimensional skin-friction coefficient on the plate y=0 increases with the increase of magnetic parameter M and it decreases with the increase of the elasticity factor in comparison with that of Newtonian fluid flow for U=0. But fig-7 shows an opposite trend of the non-dimensional skin-friction coefficient on the plate y=0 when the amplitude of the velocity of the moving plate is non-zero (U=1).



Fig-8: α =.05; E=.01;w=3;U=0;A=1;S=.1; t=.1;R=.1;Kr=.1;Sr=.1;Sc=.1;P=3;



Figure 8 shows a diminishing trend of the non-dimensional skin-friction coefficient on the plate y=1 against the magnetic parameter M when the amplitude of the velocity of the moving plate is zero (U=0). It also illustrates that the viscoelasticity factor enhances the non-dimensional skin-friction coefficient against the magnetic parameter M. Figure 9 explains the nature of the non-dimensional skin-friction coefficient on the plate y=1 against the magnetic parameter M when the amplitude of the velocity of the moving plate is non-zero (U=1). In this case, the non-dimensional skin-friction coefficient has an accelerating trend against the magnetic parameter in both Newtonian and non-Newtonian cases. Fig-9 also shows that visco-elastic factor enhances the nonthe dimensional skin-friction coefficient in comparison with that of Newtonian fluid flow.







 $\begin{array}{l} \textbf{Fig-11:} \ \alpha = .05; \ E = .01 \ ; M = 1; w = 3; U = 1; A = 1; S = .1; \\ t = .1; R = .1; K_r = .1; S_r = .1; S_r = .1; P_r = 3; \end{array}$

Figures 10 and 11 illustrate the nature of the temperature field against y, the width of the channel in cases when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases, the temperature field enhances throughout the channel against y but diminishes with the increase in the visco-elastic parameter K_1 in comparison with that of Newtonian fluid flow.









 $\begin{array}{l} \textbf{Fig-13:} \ \alpha = .05; M = 1; \ E = .01; w = 3; U = 1; A = 1; S = .1; \\ t = .1; K_r = .1; S_r = .1; S_c = .1; P_r = 3; y = .5; \end{array}$

Figure 12 and 13 describes the temperature field against the radiation parameter R in the cases when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases, the temperature field first increases and then decreases with the increase of the radiation parameter R .In both the cases the elasticity factor diminishes the temperature field in comparison with that of Newtonian fluid flow.



Figures 14 and 15 explain the nondimensional rate of heat transfer in terms of Nusselt number N_{u_0} at the plate y=0 against the magnetic parameter M when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. Both the figures show that the visco-elasticity factor gradually diminishes the non-dimensional rate of heat transfer N_{u_0} in comparison with that of Newtonian fluid flow. The figures also reveal that N_{u_0} gradually decreases with the increase of the magnetic parameter M.





 $\begin{array}{l} \textbf{Fig-17:} \ \alpha = .05; M = 1; \ E = .01; w = 3; U = 1; A = 1; S = .1; \\ t = .1; K_r = .1; S_r = .1; S_c = .1; P_r = 3; y = 0; \end{array}$

Figures 16 and 17 describe the nondimensional rate of heat transfer in terms of Nusselt number N_{u_0} at the plate y=0 against the radiation parameter R when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases, the non-dimensional rate of heat transfer N_{u_0} first increases and then decreases to a remarkable state with the increase of the radiation parameter R. Both the figures show that the visco-elasticity factor diminishes the non-dimensional rate of heat transfer N_{u_0} in comparison with that of Newtonian fluid flow.

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Figures 18 and 19 explain the nondimensional rate of heat transfer in terms of Nusselt number N_{u_1} at the plate y=1 against the magnetic parameter M when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. Both the figures show that the visco-elasticity factor enhances the nondimensional rate of heat transfer N_{u_1} in comparison with that of Newtonian fluid flow. A growing trend of the non-dimensional rate of heat transfer N_{u_1} is also observed in the case of U=0 with the increase of the magnetic parameter M.







Fig-21: $\alpha = .05; M = 1; E = .01; W = 3; U = 1; A = 1; S = .1; t = .1; K_r = .1; S_r = .1; S_c = .1; P_r = 3; y = 1.$

Figures 20 and 21 illustrate nature of the non-dimensional rate of heat transfer in terms of Nusselt number N_{u_1} at the plate y=1 against the radiation parameter R when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases, the non-dimensional rate of heat transfer N_{u_1} suffers a diminishing trend against the radiation parameter R but an opposite trend is observed against the visco-elastic parameter K_1 in comparison with that of Newtonian fluid flow.







Figures 22 and 23 explain the behavior of the concentration field against the chemical reaction parameter K_r when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. Both the figures reveal a diminishing trend of the concentration field against the chemical reaction parameter K_r but an opposite trend is found against the visco-elastic parameter K_1 in comparison with that of Newtonian fluid flow.





 $\begin{array}{l} \textbf{Fig-25:} \ \alpha = .05; \ E = .01 \ ; M = 1; w = 3; U = 1; A = 1; S = .1; \\ t = .1; R = .1; K_r = .1; y = .5; S_c = .1; P_r = 3; \end{array}$

Figures 24 and 25 describe the behavior of the concentration field against the Soret number S_r when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases, the Soret Number diminishes the concentration field while the elasticity factor enhances the concentration field in comparison with that of Newtonian fluid flow.





Figures 26 and 27 explain the nondimensional rate of mass transfer in terms of Sharewood number S_{h_0} at the plate y=0 against the chemical reaction parameter K_r when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases the non-dimensional rate of mass transfer S_{h_0} decreases with the chemical reaction parameter K_r. It is also observed that the visco-elasticity factor enhances the non-dimensional rate of mass transfer S_{h_0} in comparison with that of Newtonian fluid flow.









Fig-29: α =.05; E=.01; M=1;w=3; U=1; A=1; S=.1 t=.1; R=.1; K_r=.1; y=0; S_c=.1; P_r=3;

Figures 28 and 29 show the nondimensional rate of mass transfer in terms of Sharewood number S_{h_0} at the plate y=0 against the Soret number S_r when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. Both the figures reveal that the non-dimensional rate of mass transfer S_{h_0} decreases with the Soret number S_r . But the viscoelasticity factor enhances the non-dimensional rate of mass transfer S_{h_0} in comparison with that of Newtonian cases.







Figures 30 and 31 explain the nondimensional rate of mass transfer in terms of Sharewood number S_{h_1} at the plate y=1 against the chemical reaction parameter K_r when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases the non-dimensional rate of mass transfer S_{h_1} enhances with the growth of the chemical reaction parameter K_r. But the visco-elasticity factor diminishes the non-dimensional rate of mass transfer S_{h_1} in comparison with that of Newtonian fluid flow.

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Fig-32: α=.05; E=.01 ;M=1;w=3;U=0;A=1;S=.1; t=.1;R=.1;K_r=.1;y=1;S_c=.1;P_r=3;



 $t=.1;R=.1;K_r=.1;y=1;S_c=.1;P_r=3;$

Figures 32 and 33 illustrate the nondimensional rate of mass transfer in terms of Sharewood number S_{h_1} at the plate y=1 against the Soret number S_r when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. In both the cases, a growing trend of the non-dimensional rate of mass transfer S_{h_1} is observed against the Soret number S_r . But a diminishing trend in the non-dimensional rate of mass transfer S_{h_1} is observed against the viscoelasticity factor in comparison with that of Newtonian cases.

CONCLUSION

V.

An analysis to find out the visco-elastic effects on MHD flow with heat and mass transfer of Walters liquid (Model B') through a porous medium bounded by two infinite horizontal parallel porous plates in presence of radiation, chemical reaction, Soret effect and heat source has been presented when one plate is kept at rest while the other is oscillating in its own plane for different values of visco-elastic parameter K_1 in combination of other flow parameters. The visco-elastic effect is characterized by the non zero values of the non-Newtonian parameter K_1 whereas K_1 =0 represents the Newtonian fluid flow phenomenon.

From this study, we make the following conclusions:

- The effect of the visco-elasticity factor in the velocity field is significant and the velocity field enhances with the growth of the visco-elasticity factor.
- The temperature field is considerably affected by the variation of visco-elastic parameter and it decreases with the increase of the viscoelastic parameter.
- The effect of visco-elastic parameter is prominent in the concentration field and it increases with the increase of the visco-elastic parameter.
- The increasing values of chemical reaction parameter and Soret number decelerate the concentration field.
- The skin-friction coefficient is considerably affected by the non-Newtonian parameter on both the plates. It decreases on the plate y=0 and increases on the plate y=1 when the amplitude of the velocity of the moving plate is zero (U=0). But it increases on both the plates when the amplitude of the velocity of the moving plate non-zero (U=1) with growing effect of the non-Newtonian parameter.
- The skin-friction coefficient is also significantly affected by the magnetic parameter on both the plates. It increases and decreases on the plate y=0 against the magnetic parameter when the amplitude of the velocity of the moving plate are zero (U=0) and non-zero (U=1) respectively. But an opposite trend is observed on the plate y=1.
- The rate of non-dimensional heat transfer in terms of Nusselt number on the plate y=0 diminishes with the growth of the visco-elastic parameter and magnetic field M as well. But it exhibits mixed types of behavior against the radiation parameter. It first increases up to some value of the radiation parameter and then decreases.

- The rate of non-dimensional heat transfer in terms of Nusselt number on the plate y=1 increases with the growth of the visco-elastic parameter and decreases with the radiation parameter. But it increases against the magnetic field for U=0 and for U=1, it first increases up to some value of the magnetic parameter and then decreases.
- The rate of non-dimensional mass transfer in terms of Sharewood number on the plate y=0 enhances with the growth of the visco-elastic parameter but diminishes with the growing effect of both Chemical reaction parameter and Soret number. An opposite trend is observed for the rate of non-dimensional mass transfer in terms of Sharewood number on the plate y=1.

REFERENCES

- Abel, M. S., P. G. Siddheshwar, and M. M. Nandeeppanavar (2007). Heat transfer in a visco-elastic boundary layer flow over a stretching sheet with viscous dissipation and non-uniform heat source. *Int. J. Heat and Mass Transfer*. 50(5-6), 960-966.
- [2]. Ahmed N. and Barman D MHD Oscillatory flow and heat transfer trough a porous medium bounded by two horizontal plates in presence of a heat source, Bulletin of the Gauhati University Mathematics Association, 11, 101-120, (2010).
- [3]. Ahmed, S. (2010). Influence of chemical reaction on transient MHD free Convective flow over a vertical plate in slip-flow Regime. *Emirates Journal for Engineering Research*. 15(1), 25-34.
- [4]. Alfven H. Discovery of Alfven waves, Nature 150,405, (1942).
- [5]. Angirasa, D., G. P. Peterson, and I. Pop (1997). combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium. *Int. J. Heat Mass Trans.* 40(12), 2755-773.
- [6]. Anjalidevi, S. P. and R. Kandasamy (2000). Effects of a chemical reaction heat and mass transfer on MHD flow past a semi infinite plate. Z. Angew. Math. Mech. 80, 697-701.
- [7]. Al-Odat, M. Q. and Al-Azab (2007). Influence of chemical reaction on transient MHD free convection over a moving vertical plate. *Emirates J. Engg. Res.* 12(3), 15-21.
- [8]. Attia H.A., Kotb N.A. MHD flow between two parallel plates with heat transfer Acta Mechanica 117, 215-220, (1996).

- [9]. Bodosa G.,Borkakoti A.K. Unsteady free convection MHD flow and heat transfer of a visco-elastic fluid past between two heated horizontal plates with heat sources or sinks, Indian Jour. of Theo. Phys. 51(1), 39-46, (2003).
- [10]. Bala, P., A. Reddy, N. B. Reddy, and S. Suneetha (2012). Radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source. *JAFM*, 5(3), 119-126.
- [11]. Baoku, I. G., C. I. Cookey, and B. I. Olajuwon (2012). Influence of thermal radiation on a transient MHD Couette flow through a porous medium. *JAFM*, 5(1), 81-87.
- [12]. Basu, B., V. R. Prasad, and N. B. Reddy (2011). Radiation and mass transfer effects on transient free convection flow of dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. *JAFM*, 4(1), 15-26.
- [13]. Brinkman H.C. A calculation of the Viscous Force exerted by a flowing fluid on a dense swarm of particles. Appl. Sci. Res. A1, 81-86, (1947).
- [14]. Choudhury R.C., Jha Abhay Kumar and Dhayal Sawai Singh (2006), Heat transfer to MHD flow in a horizontal channel with heat source, J Rajasthan acad. Phy. Sci. 5(2), 233-238.
- [15]. Chaudhary, R. C. and A. K. Jha (2008). Effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. *Applied Mathematics* and Mechanics. 29(9), 1179-1194.
- [16]. Choudhury, R. and D. Dey (2010). Free convective visco-elastic flow with heat and mass transfer through a porous medium with periodic permeability. *Int. J. Heat and Mass Transfer (Elsevier).* 53, 1666-1672.
- [17]. Choudhury, R. and M. Mahanta. (2009). Free convective MHD flow of a viscoelastic fluid past an infinite vertical channel. *Int. J. Appl. Math.* 23, 189-203.
- [18]. Choudhury, R. and S.K. Das. (2012) "Visco-elastic effects on free convective flow confined between a long vertical wavy wall and a parallel flat wall of equal transpiration." Int. J. of Appl. Engg. Research, Vol. 7, No. 7, pp. 691-706.
- [19]. Choudhury, R. and S.K. Das. (2012) "Visco-elastic free convective flow and heat transfer past an oscillating porous plate in the slip flow regime." Adv. in

Theo. and Appl. Math., Vol. 7, No. 4, pp. 403-416.

- [20]. Choudhury, R. and S.K. Das. (2013) "Visco-elastic unsteady MHD flow between two horizontal parallel plates with hall current."J. of Math., Vol. 5, No.1, pp. 20-28. doi 10.9790/5728-0512028.
- [21]. Choudhury, R. and S.K. Das. (2014) "Visco-elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer." J. Appl. Fluid Mech. (JAFM), Vol.7, No. 4, October, pp. 603-609.
- [22]. Choudhury, R. and S.K. Das. (2013) "Mixed convective visco-elastic MHD flow with ohmic heating." Int. J. of Comp. Appl., Vol.68, No.10, pp. 7-13,Factor:0.814(2011), doi10.5120/11613-7004
- [23]. Choudhury, R. and S.K. Das. (2013) "Visco-elastic mhd oscillatory horizontal channel flow and heat transfer with heat source." Appl. Math. and Comp. Sci., Vol. 4, No. 3, pp. 149-206.
- [24]. Choudhury, R. and S.K. Das. (2012) "Visco-elastic MHD fluid flow over a vertical plate with Dufour and Soret effect." Int. J. of Sci. and Engg. Res., Vol. 4, No. 7 pp.11-17. Impact Factor: 1.4(2011), ICV (Index Copernicus Value) 2012: 5.27.
- [25]. Choudhury, R. and U. Das (2012). Heat transfer to MHD oscillatory visco-elastic flow in a channel filled with porous medium. *Physics Research International*. 2012,Ar ID 8795537, 5 pages.
- [26]. Cowling T.G. Magneto hydrodynamics, Wiley Inter Science, New York (1957).
- [27]. Cramer K.R. and Pai S.I. (1973), Magneto Fluid dynamics for Engineers and applied physicists, McGraw-Hill, New 5. Cramer K.R. and Pai S.I. (1973), Magneto Fluid dynamics for Engineers and applied physicists, McGraw-Hill, New York.
- [28]. Devi, S. P. A., and B. Ganga (2010).Dissipation effects on MHD non linier flow and heat transfer past a porous surface with prescribed heat flux. *JAFM*, 3(1), 1-6.
- [29]. El-Aziz, M. A. (2009). Radiation effect on the flow and heat transfer over an unsteady stretching sheet. *International Communications in Heat and Mass Transfer*. 36(5), 521- 524.
- [30]. Elbashbeshy, E. M. A., D. M. Yassmin, and A. A. Dalia (2010). Heat transfer over

an unsteady porous stretching surface embedded in a porous medium with variable heat flux in the presence of heat source or sink".*African Journal of Mathematics and Computer Science Research.* 3(5), 68-73.

- [31]. Ferraro V.C.A. and Plumton C. An introduction to Magneto Fluid Mechanics, Clarandon Press, Oxford (1966).
- [32]. Kandasamy, R., K. Periasamy, and K. K. S. Prabhu (2005). Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. *Int. J. Heat and Mass Transfer.* 48, 21-22, 4557-4561.
- [33]. Kelly, D., K. Vajravelu, and L. Andrews 1999. Analysis of heat mass transfer of a visco-elastic, electrically conducting fluid past a continuous stretching sheet. *Nonlinear Analysis: Theory, Methods and Applications.* 36(6), 767-784.
- [34]. Makinde O.D., Mhone P.Y. Heat transfer to MHD oscillatory flow in a channel filled with porous medium, Rom. Jour. Phys., 50,931-938, (2005).
- [35]. Muthucumaraswamy, R. and P. Chandrakala, (2006). Radiation heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. *J. of Appl.Mech. and Engng.* 11(3), 639-646.
- [36]. Muthucumaraswamy, R. and S. Meenakshisundaram (2006).Theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature. *Theoret. Appl. Mech.* 33(3), 245-257.
- [37]. Nigam S.D. and Singh S.N. Heat transfer by Laminar flow between parallel plates under the action of transverse magnetic field, Q.J. Mech. Appl. Math., 13, 85-87, (1996).
- [38]. Rao, J. A., S. Sivaiah, and R. S. Raju (2012). Chemical effects on an unsteady MHD free convection fluid past a semiinfinite vertical plate embedded in a porous medium with heat absorption. *JAFM*, 5(3), 63-70.
- [39]. Raptis A., Massailas C. and Tzivandis G. Hydro magnetic free convection flow through porous medium between two parallel plates, Phys. Lett. 90(A), 288-289, (1982).
- [40]. Reddy, M. G. and N. B. Reddy (2011). Mass transfer and heat generation effects on MHD free convection flow past an

inclined vertical surface in a porous medium. *JAFM*, 4(2), 7-11.

- [41]. Sandeep, N., A. V. B. Reddy, and V. Sugunamma (2012). Effect of Radiation and Chemical Reaction on Transient MHD Free Convective Flow over a Vertical Plate Through Porous Media. *Chem. and Process Engng Res.* 2, 1-9.
- [42]. Sharma, P. K. (2005). Fluctuating thermal and mass diffusion on unsteady free convective flow past a vertical plate in slip-flow regime. *Latin American Applied Research.* 35, 313- 319.
- [43]. Shercliff J.A. A text book of Magneto hydrodynamics, Pergamon Press, London. (1965).
- [44]. Singh, P., N. S. Tomer, and D. Sinha (2010a). Numerical study of heat transfer over stretching surface in porous media with transverse magnetic field. *Proceeding of International Conference* on Challenges and application of Mathematics in Sciences and Technology. 422-430.
- [45]. Singh, P., N. S. Tomer, S. Kumar, and D. Sinha (2010b). MHD oblique stagnationpoint flow towards a stretching sheet with heat transfer. *International Journal of Applied Mathematics and Mechanics*. 6(13), 94-111.
- [46]. Singh, P., A. Jangid, N. S. Tomer, S. Kumar, andD. Sinha (2010c). Effects of Thermal Radiation and Magnetic Field on Unsteady Stretching Permeable Sheet in Presence of Free Stream Velocity.*Int. J.of Information and Mathematical Sci.* 6(3), 63-69.
- [47]. Sonth, R. M., S. K. Khan, M. S. Abel, and K. V. Prasad (2002). Heat and mass transfer in a visco-elastic flow over an accelerating surface with heat source/sink and viscous dissipation. *Heat and Mass Transfer*. 38(3), 213-220.
- [48]. Soundalgekar V.M. and Bhatt J.P. Oscillatory MHD channel flow and heat transfer, Indian J. Pure Appl. Math., 15,819-983, (1971).
- [49]. Subhash, A. M., A. Joshi, and R. M. Sonth (2001). Heat transfer in MHD viscoelastic fluid flow over a stretching surface. *Zeitschrift f r angewandte Mathematikund Mechanik.* 81, 10, 691-698.
- [50]. Suneetha, S., N. B. Reddy, and V. R. Prasad (2010). Radiation and mass transfer effects on MHD free convective dissipative fluid in the presence of heat source/sink. JAFM, 4(1), 107-113.

- [51]. Vajravelu K. An exact periodic solution of a hydro magnetic flow in a horizontal channel, ASME journal of Applied Mechanics, 55, 981-983, (1988).
- [52]. Walters, K. (1960). The motion of an elastico-viscous liquid contained between co-axial cylinders (II). *Quart. J. Mech. Appl. Math.* 13, 444-461.
- [53]. Walters, K. (1962). The solution of flow problems in the case of materials with memories. *J. Mecanique.* 1, 473-478.